Seminar Paper

Convertible Bonds: An Overview for Investors and Issuers.*

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Abstract:

This paper offers an overview of the distinctive features of convertible bonds for both investors and issuers. For investors, it focuses on diversification potential and the derivative character of the product. It shows that reasonable potential exists when adding convertibles to bonds and stocks. For issuers, the paper presents the academic justification for convertible bonds, namely information related problems like the overinvestment problem, divergent risk perception or adverse selection at the issuance of conventional equity. In a third part, the paper introduces three classical pricing models (contingent claim analysis, the no-arbitrage equilibrium condition and the binomial tree approach, where the latter is discussed to some extent) and finally sketches a few of the more recent developments in this field of research.

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Convertible bonds have gained in popularity in the last few years, especially because of the generally bigger risk aversion of investors. They are a well established product with derivative qualities offering a certain amount of capital protection combined with upside participation.

Basically, a convertible bond is corporate debt with the additional right for the investor to convert the bond into shares. The conditions of conversion, such as the time conversion is possible, and conversion price or conversion rate are predetermined at emission of the convertible bond.

There are two forms of convertible debt: "convertible bonds" offer the investor the right to convert the paper into shares of the emitting company, whereas "exchangeable bonds" offer the right to convert the paper into shares of a third party. However, the analysis of convertible debt in most cases leads to similar results for both, hence I will concentrate on convertible bonds. In terms of finance theory, a convertible bond can be thought of as a combination of straight debt and a call option on the share, the investor taking a long position in the option.

Convertible Bonds from an investor's view

For investors, convertible debt has three interesting, distinctive features: first, they offer good diversification potential, as a portfolio of convertible bonds is highly correlated neither with stock nor with straight bond portfolios. Second, convertible debt offers through it's bond character a certain capital protection, but still leaves the investor the opportunity to participate in positive stock price developments. This feature pays especially well in times of high volatility and raised risk aversion. Third, convertible bonds reduce certain information economic problems between the emitting company and investors.

Potential of diversification

Bearing in mind that a convertible bond is technically a combination of straight debt and a call option on a company's stock, it is not immediately apparent that this instrument should offer good diversification possibilities against stocks and bonds. But convertible bonds do in fact offer a reasonably big potential of diversification if used as a third asset class together with the classical fixed income (straight bonds) and equity instruments.

I illustrate this by comparing two portfolios. To eliminate effects specific to any one company, I use indices for each of the three instruments. Straight Bonds are represented by the JP Morgan Bond Index (Europe). Stocks are represented by the MSCI (Europe) Stock Index. For convertible bonds I use the UBS Convertible Bonds index (Europe). Each of the three indices is denominated in USD. I use daily data from January 03 2000 to May 21 2004, representing 1144 data points. Returns are calculated month/month every day, leaving 1122 data points.

The following measures can be obtained:

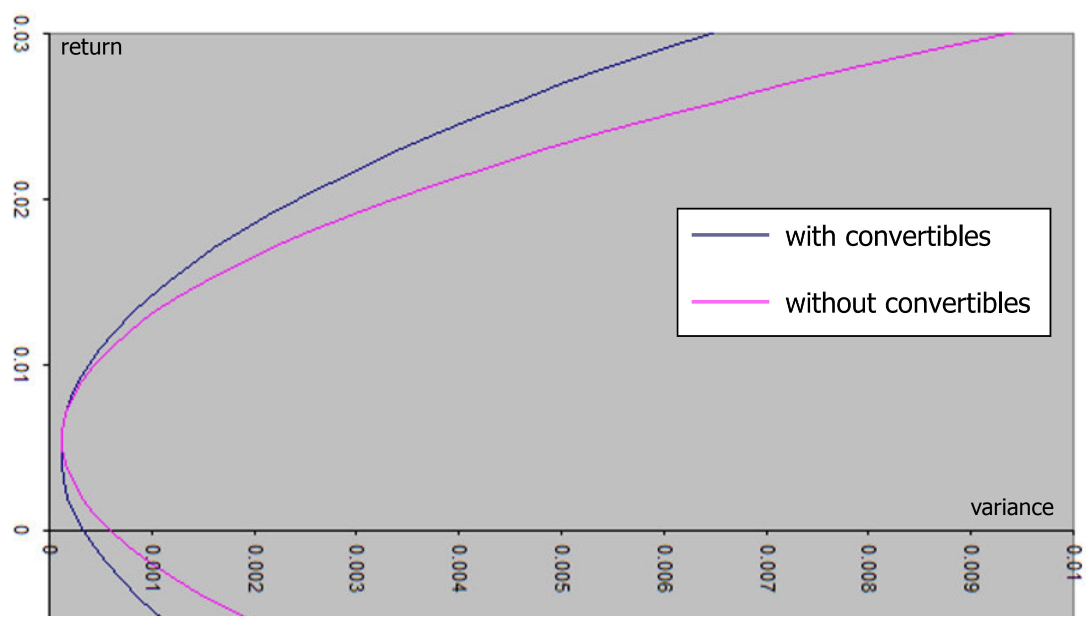
Table 1: Statistical measures of the index sample (one month returns)

Instrument:	*Variance:*	*Std. dev.:*		*Correlation:*	Bonds	Conv.	Stocks
JPM Bond Index	0.0001	0.0121		Bonds	1.0000	0.3204	-0.0891
UBS CB Index	0.0011	0.0335		Convertibles	0.3204	1.0000	0.7145
MSCI Stock Index	0.0016	0.0402		Stocks	-0.0891	0.7145	1.0000

The statistical properties of convertible bonds are, despite their name, more similar to those of stocks. This also becomes apparent from the correlation matrix, with a correlation of more than 0.7 for stocks and convertibles. However, the stability index of convertible bonds and stocks amounts to only about $R^2 = 0.51$ over the whole sample. This leaves enough potential for diversification by adding a portfolio of convertibles to bonds and stocks.

Given the data set, it is easy to determine variance-covariance matrices and expected returns for both portfolios with and without convertible debt. Using plain vanilla mean variance portfolio theory, it is possible to construct an efficient frontier for both these portfolios.

It becomes apparent, that convertible bonds offer reasonable diversification potential, even though the return of the minimum variance portfolio is higher without adding convertible bonds to the portfolio mix.[1]



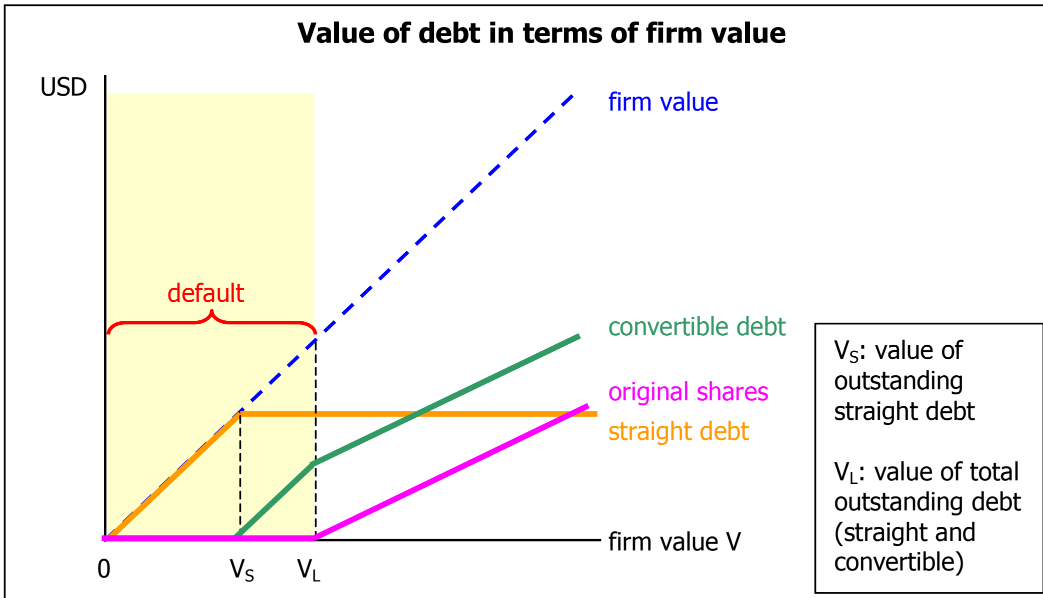
Graph 1: the efficient frontiers of portfolios with and without convertible bonds. Source: author's calculations

It is interesting that, the lower the variance, the lower the effect of adding convertibles. This may be because of the asymmetric nature of convertible bonds. In titles with very low variance, the convertible will behave much like a bond, as it's option component will be close to worthless. Therefore, it will not offer significant diversification potential if added to a broadly diversified mixture of bonds and stocks. However, the more variance there is in the market, the more valuable the option component becomes, thus giving the convertible bonds portfolio a price statistic different from both bonds and stocks and therefore widening the feasible investment set for the investor.

[1] see appendix, equations 11 and 12.

Capital protection

I offer a simplified model of a firm with straight debt, convertible debt and shares outstanding as only assets. In this model, the firm stops business immediately after maturity of the convertible bond.[2] For every existing share at this point in time, another share is issued, but only if conversion takes place. This leads to a dilution of 50% (this is the reason why the value lines of shares and convertible bond rise at only half the slope of straight debt: ceteris paribus for every dollar in firm value above V_L, 50% goes to original shareholders, 50% goes to new shareholders, who got their share from conversion of a bond). V_L denotes the value of all outstanding debt, convertible and straight. If V is smaller than V_L, the company is unable to pay back it's debt and defaults. In that case, first the holders of straight debt are paid out. V_S denoting the value of outstanding straight debt, holders of convertible bonds have no capital protection below V_S. Between V_S and V_L, the capital of convertible debt holders is paid back. Only above V_L, shareholders participate in the value of the firm. However, holders of convertible debt may choose to convert if firm value is above V_L, deluding the value of each share.



Graph 2: Value of debt in terms of firm value.

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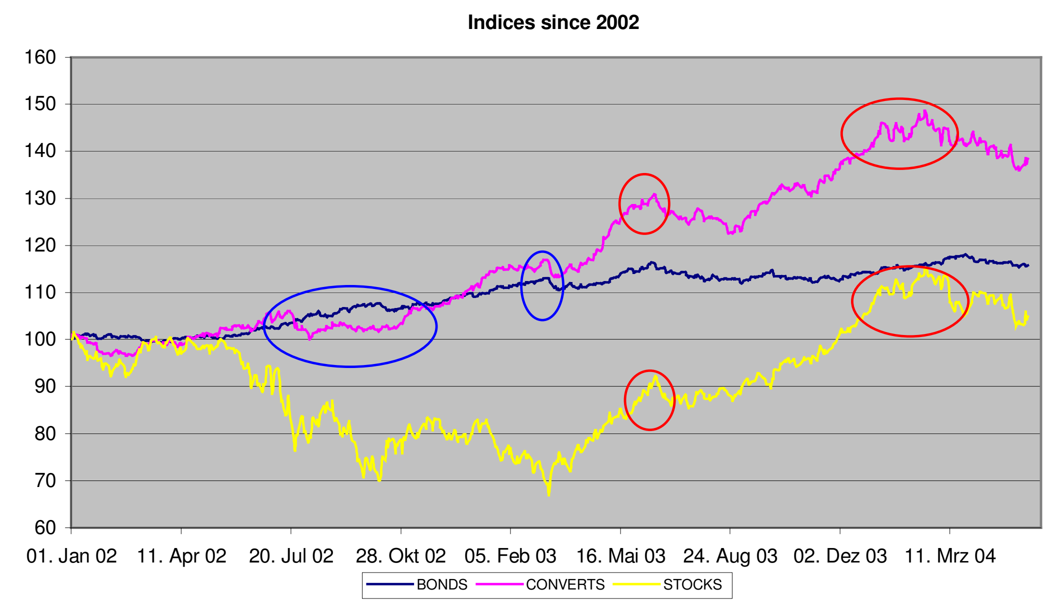
[2] This condition might not actually be necessary, but it clarifies the argumentation.

Obviously, this model strongly simplifies the problem. First and foremost, it is a strong assumption that the firm goes out of business immediately after maturity of the debt, as is the structure of the firm itself. Also, it appears not very realistic that any firm should have convertible debt outstanding equal to its equity – however, this only influences the dilution effect, i.e. the slope of the curve beyond V_L.

With all these flaws, the model still shows the basic advantages of convertible bonds versus both straight debt and shares. Convertible bonds offer better capital protection than shares (namely between V_S and V_L) and better upside participation than straight debt (which do not offer any).



Graph 3: JP Morgan Bond Index Europe, UBS Convertible Bond Index Europe and MSCI Europe Stock Index, from 01-2002 to 05-2004, indexed January 2002. Source: author's calculations

The effect of this asymmetrical payoff can be illustrated very well by showing indices for stocks, bonds and convertibles from January 2002 through May 2004, a period of sideways or bearish stock markets in the first part of the period and bullish stock markets from early 2003 onwards. Through the sideways and downward stock market, the convertible bond index follows closely the bond index. But from around March 2003, when stock markets turn bullish again, the convertible bond index stops following the bond index and instead follows the pattern of the stock market.

In the graph shown, the bonds indicator is the JP Morgan Bond Index for Europe, the convertibles (converts) indicator is the UBS Convertible Bond Index for Europe, and the stocks index is the MSCI Stock Index for Europe. The blue circles indicate where the convertible bond index reacted on a movement in bond markets (or failed to react on movements in the stock market), whereas the red circles indicate the reverse.

This intuitive notion is supported by statistical figures. Table 2 shows the correlations for the two periods and the stability indices.

Table 2: Correlation and stability indices for subperiods 02-03 and 03-04						
Correlation:						
01.Jan.02 – 12.Mar.03	UBS CB Index	MSCI Stock Index		*13.Mar.03 – 21.May.04*	UBS CB Index	MSCI Stock Index
JPM Bond Index	0.8844	-0.9203		JPM Bond Index	0.7945	0.7653
UBS CB Index		-0.6819		UBS CB Index		0.9832
Stability indices:						
01.Jan.02 – 12.Mar.03	UBS CB Index	MSCI Stock Index		*13.Mar.03 – 21.May.04*	UBS CB Index	MSCI Stock Index
JPM Bond Index	0.78	0.85		JPM Bond Index	0.63	0.59
UBS CB Index		0.46		UBS CB Index		0.97

The stability index for Convertibles/Bonds drops from 0.78 to 0.63 (almost 20%), whereas the stability index for Convertibles/Stocks rises from 0.46 to 0.97.

Overinvestment problem

A firm may have two investment opportunities in prospect, one needs to be funded immediately and will pay off at the end of the first period, the second project will need funds only at the end of the first period with pay off at the end of the second period. In this case, the firm has three debt financing alternatives. It can issue two-period straight debt at the beginning of the first period, it can issue twice one-period straight debt at the beginning of each period, or it can issue a convertible bond with maturity after one period. For the firm, the best alternative would be to issue a two-period straight bond, as it both ensures funding for the second project and minimizes issuance cost. As Mayers (1998) notes, "[t]he problem with this financing alternative is that managerial discretion and the set of available investments insure that a second-period project will be financed, whether the investment option turns out to be valuable or not."[3]

Convertible debt gives the investor some disciplinary power over management. Should it turn out management did not assess investment prospects well or should other, more behavioural issues arise, investors can choose to demand redemption. Thus, convertible debt equips investors with shares combined with an European stile exit option. The option premium paid for this option corresponds to the premium paid on the convertible shares.

A similar point is made by Jensen (1986). He concentrates on free cash flow as all cash flows in excess of those required to fund all projects that have positive net present value. Jensen states that "[b]y issuing debt in exchange for stock, managers are bonding their promise to pay out future cash flows […] Thus [convertible] debt reduces the agency costs of free cash flow […]"[4] This argument also explains the empirical fact that the call of convertible bonds and the (forced) conversion reduces stock prices[5]. By forcing conversion, investors lose part of their disciplinary power, which in turn raises agency costs.

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[3] Mayers (1998), pp. 86

[4] Jensen (1986), pp. 324

[5] See e.g. Jen, Choi and Lee (1997), Pilotte (1992) or Mikkelson and Partch (1986)

Convertible bonds from an issuer's view

Free lunch?

Convertible bonds are a relatively old financial instrument, compared to options and other derivatives. Nonetheless, they proved difficult to explain in academic analysis for decades, as they seemed to contradict the efficient market hypothesis. The question was, why firms should offer convertible bonds which are nothing but a simple combination of straight debt and a call option on equity. Or put differently: why "should sophisticated investors be willing to pay more [... for convertible bonds] than for separate offerings of straight debt and straight equity?"[6] Until the late 1970s, this fact was explained with a certain delusion held by corporate treasurers and financial advisors. But this was obviously an unsatisfying result: because convertible bonds survived for so long, the price for them had to be justified, otherwise market forces would have erased the mispricings long ago – otherwise market mechanisms indeed performed very poorly.

This meant that there are additional costs for the issuer. It was unclear, however, what these costs were. In 1981, Brennan and Schwartz offered a first possible explanation, namely different risk assessments by issuer and investor for debt. Another one is the implicit exit option mentioned in the previous chapter. I will quickly introduce the most important of these concepts.

Divergent risk perception

The higher investors estimate the risk of an investment, the more return they demand: the market demands a premium for taking additional uncertainty. Especially for young, rapidly growing corporations it can be very difficult to convince investors about the level and

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[6] Brennan and Schwartz (1981), pp. 163

stability of the company's risk, forcing the company to bear interest costs which are higher than what management would expect and warrant. In this case, costs of straight debt may become prohibitively high for the company.

Remember that convertible bonds technically are similar to straight debt with call options. Companies with higher risk, operational or financial, tend to have more volatile stock prices. Increases in risk will reduce the value of the debt. But the option component of the convertible gains in value with increasing volatility of the underlying stock (following the standard Black and Scholes option pricing approach). This is one reason, why convertible debt costs less interest than straight debt: the effects of the straight debt and the option component of the convertible bond cancel each other out to a certain degree, making the instrument as a whole less risk sensitive.

Brennan and Schwartz (1981) offer a short numeric example of the negotiation effect this risk insensitivity has. Imagine that the market for straight debt demands 14% interest for firms with medium risk, but 16% for firms with high risk. Coupon rates on convertible debt amount to 11% for a company with medium risk, to 11.25% for a company with high risk. Now further suppose that management believes the company to be one of medium risk, but which the markets perceives as high risk. The market would demand 2% more interest rate on straight debt than other companies the management deems comparable risk. Although convertible debt also appears expensive, the difference amounts to only 0.25%. Thus, the effect of the divergence in risk assessment between the companies management and the public in the market is far less for convertible debt.[7]

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[7] The example follows Brennan and Schwartz (1986), pp. 166f.

Convertible bonds as backdoor equity financing

Stein (1992) develops yet another rationale for firms to issue convertible debt. As was shown by Myers and Majluf (1984), conventional equity issues can be unattractive because of informational asymmetries between management and investors. In this case, the issue of convertibles can help mitigating these problems. Stein also points out several partial explanations for issuing convertibles which were examined later in more detail for example by Jen, Choi and Lee (1997) or by Mayers (1998).

Stein develops a model similar to this of Myers and Majluf. The model contains three firm types ("bad", "medium" and "good") and lasts three time periods (0, 1, 2). The model shows that "the availability of convertible bonds makes it possible to sustain a separating equilibrium in which all types of firms issue fairly priced claims and invest efficiently. This outcome is not generally possible when long-term debt and equity are the only financing options."[8] The model also shows, that bad firms will never issue long-term debt and will issue convertible debt only if the possibility of overpricing is sufficiently high and the cost of financial distress sufficiently low. It follows, that convertibles would be especially valuable for firms that either suffer from significant problems related to information asymmetries, would face big costs in case of financial distress, or both.

Both these problems for the company are addressed by using convertible debt when issued with a call provision. On the one hand, the convertible reduces the problem of asymmetric information,[9] on the other hand the convertible mitigates adverse selection problems of issuing equity as described by Myers and Majluf (1984). Thus, the convertible can serve as an indirect mechanism for implementing equity financing. Although the outcome may be somewhat uncertain, firms with good prospects will "deliberately" issue convertible debt with the intention of changing it into equity at some later point in time.

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[8] Stein (1992), pp. 6.

[9] as shown by Brennan and Schwartz (1986) and earlier in this paper.

Sequential Investments

A new approach to why firms issue convertible bonds was introduced by Mayers (1998). In his argumentation, Mayers assumes a sequential financing problem, that is a project with immediate need for funds plus an investment option with a future maturity date. As explained with the overinvestment problem, raising funds up front for both the initial project and the investment option produces a principal-agent problem between the manager and the investor, whereas issuing twice single period straight debt would produce high costs. Bhagat and Frost (1986) show evidence of economies of scale in issuance costs, so companies should avoid multiple small issues of debt or equity. By issuing convertible debt, managers have the possibility to save some of the costs of issuing debt, while still avoiding agency costs arising from the overinvestment problem, which in the end would probably have to be borne by the agent (i.e. the company) all the same[10], given the efficiency of capital markets.

Mayers (1998) also explains the role of the call provision, that is the option for the issuer to force conversion of the convertible before maturity: once management discovers that the investment opportunity for the second period is valid, it will force conversion, which allows the firm to proceed with its financing plan. He finds evidence that after conversion, together with the new equity, companies tend to raise new debt as well: he finds "strong evidence of incremental investment activity commencing in the year of the call [of the convertible] and continuing for the following three years. […] The sequential-financing hypothesis provides an explanation for these joint increases: the evidence supports the conclusion that convertible provision are designed to facilitate the future financing of valuable real investment options."[11]

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[10] Jensen and Meckling (1976), pp. 324.

[11] Mayers (1998), pp. 85.

Growth Funding Hypothesis

In their paper from 1997, Jen, Choi and Lee offer a new rationale for firms to issue convertible debt: the growth funding hypothesis states that firms operating in high-growth markets require much capital to fund investment opportunities, but have only limited additional debt capacity. Convertible bonds have the additional advantage to provide a gradually growing equity base. Moreover, convertible bonds are likely to both reduce flotation costs as well as information costs compared to conventional equity offerings.[12]
The second rationale mentioned by the authors is the "expected costs of financial distress hypothesis" mentioned already by Stein (1992), but they extend it somewhat. Because convertible bonds pay lower interest and because in case of conversion the face value has not to be paid back, convertible debt reduces both the probability and the expected cost of financial distress (i.e. illiquidity and, ultimately, default). This is particularly important for small firms in rapidly growing markets. Such companies are generally considered risky and therefore have to pay higher interest rate in the first place. Typically, such companies also have comparably small liquidity reserves, and the expected cost of financial distress is high, because much of the firm value is non fungible.
In their article, Jen et al state that "the market is likely to respond most favorably to convertibles when the firm appears to have high expected costs of financial distress or, alternatively, to have limited additional debt capacity."[13] And indeed, they find that the market reaction for firms with a low rating by Standard and Poors' is less negative to announcements of convertible bond offerings.

[12] Jen, Choi and Lee (1997), pp. 49.

[13] Jen, Choi and Lee (1997), pp. 50.

Pricing

In theory, convertible bonds are not very hard to price, as they offer a classical example of contingent claim analysis introduced by Merton (1974). But convertible bonds traded in the market often have additional features such as call options for the issuer, giving him the right to force conversion prior to maturity, or put options for the investors, giving them the right to demand redemption before maturity. In the past thirty years, several models for pricing convertible bonds have been proposed.

Contingent claim analysis

Ingersoll (1977) uses the Black-Scholes option pricing model to valuate the convertible bond as a contingent claim on the firm as a whole. He distinguishes between straight debt, non-callable convertibles and callable convertibles.

Ingersoll gains some very interesting insights into pricing and valuation of convertible debt. For example, he shows that for a non-callable convertible issue it is never optimal to convert before maturity. The same holds for callable convertibles, if no call takes place. If the convertible is called, however, Ingersoll shows that it can never be optimal to convert until such action is forced by a call. He summarizes these results as: "The right to exchange a convertible security for the underlying common stock should never be voluntarily exercised."[14] He concedes, though, that there exist some situations, were voluntary conversion is optimal, i.e. immediately prior to an adverse change in the conversion rights, meaning decreases in the convertibility of the bond or an increase in the conversion price. Finally, Ingersoll deploys the optimal call strategy for the company: it should call a convertible at the point when its conversion value equals the call price. Unfortunately,

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[14] Ingersoll (1977), pp. 298.

though, he states that "even casual empiricism indicates that the implied strategy [… of optimal call policy] is completely at odds with the observed practice of firms."[15]

Equilibrium pricing and no arbitrage condition

Building on the work of Ingersoll, Brennan and Schwartz (1980) propose an analytical approach for pricing convertible bonds. They split up the value of a convertible into two parts: the straight debt part and the conversion privilege. The value of the former depends upon the coupon rate and maturity as well as the risk of default, and upon the overall level of interest rates. The value of the conversion privilege is modelled to be a function of the risk and capital structure of the firm, it's payout and call policy and the conversion terms as well as the current stock price.

The fair value of a convertible bond is the equilibrium price which offers no arbitrage potential to neither purchaser nor to short seller, given optimal conversion strategy of the bondholder and optimal call policy of the issuer.

Brennan and Schwartz take this definition and transfer it into a model containing constraints for optimal conversion, optimal call policy and maturity / bankruptcy conditions. In a two stage procedure they first formulate a differential equation and solve it for the value of outstanding senior debt given that the convertibles are no longer outstanding. In a second stage the results for outstanding senior debt can be substituted in the differential equation, making it possible to solve it for the value of the convertible.

Binomial approach

By 1998, when Tsiveriotis and Fernandes published their paper, it was apparent that, because of various terms of realistic convertible bonds, it was impossible to unbundle the bond and option components of the product and value them independently. These terms

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[15] Ingersoll (1977), pp. 320.

include the possibility of early conversion, callability by the issuer, putability by the holder and special provisions on accrued interest upon early termination, and introduce extra degrees of freedom that depend on both the equity and the fixed income parts of a convertible bond, not to mention the at least in part strong assumptions made by Ingersoll (following Black-Scholes)[16].

A key feature of this valuation model comes from the fact that any convertible bond has components of different default risk. Whereas the equity part of a convertible bond has zero default risk, because the firm can always deliver it's own stocks[17], the coupon payments as well as the face value of the bond are subject to credit risk. While the Black Scholes model can account for credit risk in general, it is not valid for pricing convertible bonds, because only a part of the value of the convertible bond is exposed to default risk, and – moreover – the size and value of this part are not known ex ante.

Tsiveriotis and Fernandes therefore choose a somewhat different approach, namely relying on the fact that "the value of the future cash payments a rational [convertible bond] holder will choose to receive is itself a derivative of the underlying equity and interest rates"[18]. This means, they model the convertible bond price as a contingent claim analysis on the underlying equity and transform it into binomial form. Hull (2003) explains the binomial approach qualitatively and gives an easy numerical example. Beginning with a binomial tree of stock prices, in every node the value of the convertible bond is calculated given a) that conversion will ultimately take place and b) that redemption will ultimately take place. The sum of these two gives the value of the convertible bond at any given node. If possible, it is optimal for the issuer to force conversion if the value of the bond exceeds its conversion

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[16] Ingersoll (1977), pp. 291ff.

[17] For exchangeable bonds this is not automatically the case.

[18] Tsiveriotis and Fernandes (1998), pp. 96.

value. The value of the convertible can then be calculated working backwards through the tree. The following example is adapted from Hull (2003)[19]:

First, a binomial stock price tree is generated. Starting with an initial stock price of $S_0 = 50$, the stock price rises with factor u = 1.1618 at a positive development and falls with the factor 1/u = 0.8607 in case of a negative development. This yields the following stock price tree over four periods:

Table 3: Binomial stock prices

t = 0	t = 1	t = 2	t = 3
			node G $s_3 = 78.41$
		node D $s_2 = 67.49$	
	node B $s_1 = 58.09$		node H $s_3 = 58.09$
node A $s_0 = 50$		node E $s_2 = 50$	
	node C $s_1 = 43.04$		node I $s_3 = 43.04$
		node F $s_2 = 37.04$	
			node J $s_3 = 31.88$

The up and down probabilities are constant p = 0.52 and q = (1 – p) = 0.48 in every node, the appropriate interest rate for bonds is i = 15% p.a., the risk free rate is r = 10% p.a. Each period lasts 3 months, so n = 0.25. The company offers a convertible bond callable over the whole period at c = 115 with face value f = 100. The conversion ratio is γ = 2:1 (i.e. investors receive two shares for every convertible bond). Thus, in period t = 3, the value of the convertible C is equal to

$$C = \max(f, \gamma \cdot s_3) \tag{1}$$

meaning that investors will either demand redemption if $\gamma \cdot s_3 < f$, or will convert otherwise. At maturity, investors can choose between redemption (i.e. receiving back face value f) or conversion (i.e. receiving γ stocks worth s_3 each). For example, at node G, the convertible is

[19] In particular, Hull uses continuous interest.

worth C = $\gamma \cdot s_3$ = 156.82. Obviously, investors will choose to convert. At node J, the stock price has fallen to s_3 = 31.88; investors will demand redemption, thus C = f = 100.

Going backward through the tree, at every node the value of the convertible is determined given ultimately either redemption or conversion will take place. The value of the convertible bond at the node equals the sum of these two conditional outcomes.

At node E, the value of the debt part B is equal to the face value paid out at redemption, multiplied with the possibility redemption will take place and discounted over one period:

$$B_t = \left((B_{t+1} \mid u) \cdot p + (B_{t+1} \mid d) \cdot q \right) \cdot \frac{1}{(1+i)^n}$$
$$B_E = (0 \cdot 0.52 + 100 \cdot 0.48) \cdot \frac{1}{1.15^{0.25}} = 46.35 \tag{2}$$

At node E, the value of the equity part E is equal to the value of the stock received (that is the stock price at conversion multiplied with the conversion ratio γ) if conversion takes place, multiplied with the possibility of conversion and discounted over one period:

$$E_t = \left((E_{t+1} \mid u) \cdot p + (E_{t+1} \mid d) \cdot q \right) \cdot \frac{1}{(1+r)^n}$$
$$E_E = (116.18 \cdot 0.52 + 0 \cdot 0.48) \cdot \frac{1}{1.10^{0.25}} = 58.99 \tag{3}$$

Note that the value of the stock is discounted with the risk free rate, because in the model equity is not subject to default risk: the firm can always issue new equity without cost.[20] The value of the convertible bond equals the sum of the bond and equity component.

As has been shown by Ingersoll,[21] it is optimal for the company to call the convertible bond as soon as its value exceed the call price, thus as soon as the condition $C \geq c$ is satisfied, the issuer should call the bond forcing conversion.

This yields:

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[20] This is not automatically the case for exchangeable bonds.

[21] Ingersoll (1977), Theorem III, pp. 299.

Table 4: Convertible bond valuation using the binomial approach			
t = 0	t = 1	t = 2	t = 3
			node G E = 156.82 B = 0 C = 156.82
		node D E = 134.08 B = 0 C = 134.08	
	node B E = 95.73 → 116.18 B = 21.48 → 0 C = 117.21 > c → 116.18		node H E = 116.18 B = 0 C = 116.18
node A E = 62.65 → 73.03 B = 42.32 → 31.54 C = 104.97 → 104.57		node E E = 58.99 B = 46.35 C = 105.34	
	node C E = 29.95 B = 68.04 C = 97.99		node I E = 0 B = 100 C = 100
		node F E = 0 B = 96.57 C = 96.57	
			node J E = 0 B = 100 C = 100

At node D, the value of the convertible lies above conversion price of 115, which would indicate a call. However, if in D conversion takes place, the company would exchange two shares worth $2 \cdot 67.49 = 134.98$ for a convertible worth C = 134.08, which is obviously not optimal for the company.

At node B, the situation is different. Again, the value of the convertible lies above the call price, which suggests conversion. And indeed, now the company can exchange two shares worth $2 \cdot 58.09 = 116.18$ for one convertible bond worth 117.21, making the company better off. This result changes the values for E, B and C in node B and thus also affects the values in node A, reducing the value of the convertible bond from 104.97 to 104.57.

Because conversion will take place in t = 2 if node B is reached, this eliminates the possibility of ever reaching note D, which in turn eliminates the somewhat pathological result from node D and preserves Ingersoll's optimal call policy.

Other pricing models

Since the binomial model by Tsiveriotis and Fernandes in 1998, several other pricing models were introduced, relying on increasingly complex math. Particularly interesting appear the models of Kariya and Tsuda (2001), Yigitbasioglu (2002) and Bermudez and Webber (2003).

Kariya and Tsuda develop a time dependent Markov model. The authors split the value of the convertible into what they call "latent bond value" and "latent CB call" and price each stochastically. Combining these two parts yields a formula for pricing simultaneously multiple convertible bonds, taking into account their correlation structure. The model is also suitable for the pricing of single convertible bonds, as evidence from the Japanese market suggests.

Another approach is proposed by Yigitbasioglu: he uses a five-factor model of convertible bond prices, following the Cox-Ross-Ingersoll model. The factors are computed using rather sophisticated models for every factor, starting from a statistical sample and using iterative and self-learning methods to determine factor values.

The paper by Bermudez and Webber concentrates on the role of default of the issuer. The authors model convertible bond prices as a two factor model, the factors being the asset value of the issuing firm and the short term riskless interest rate. What distinguishes the model by Bermudez and Webber from other models is that it allows for detailed modelling of default and recovery values.

Conclusion

Convertible bonds, although they may be considered somewhat old-fashioned, still offer good opportunities for both investors and issuers. For the investor, the derivative character of the instrument may prevail, especially in times of volatile, uncertain markets. Using not very sophisticated statistical methods it was indicated that convertible bonds offer reasonable potential for diversification and behave worthwhile in both sideways and upside markets. Meanwhile, on behalf of the issuer, convertible bonds are able to solve or at least mitigate information related problems inherent to both equity and straight debt issuance and reduce the (accounting) costs of capital. The information related problems address the overinvestment problem, divergent risk perception by issuers and investors, adverse selection of equity issuance and others.

Pricing of convertible bonds is very complicated and still an open issue in this field of research. For most basic purposes, the convertible debt can be priced using the contingent claim analysis, which leads to the no-arbitrage and the binomial approaches. But real life instruments have numerous features, which call for more sophisticated methods. The paper introduces a few of the more recent developments in this field.

Mathematical appendix

The efficient frontier has been calculated using Microsoft Excel. The first step was to adjust the three indices to the same length: the data source of the JP Morgan index contained 365 data points per year, where data on weekends and holidays simply had been copied from the preceding banking day. These double entries were deleted. This left a sample of 1145 data points from 03 January 2000 to 21 May 2004.

The calculations base on one month returns, calculated every day. The first one month interval ranges from 03 January 2000 to 03 February 2000 and contains 24 days. All subsequent months also contain 24 days, the last one ranging from 20 April 2004 to 21 May

2004. From the 1145 data points, the sample looses the last 23 entries, resulting in 1122 one month returns.

The calculation of the efficient frontier relies on simple mean variance calculation. To keep calculations easy, the weights of the three instruments are equal, thus

$$\boldsymbol{\omega} = \mathbf{1} \tag{5}$$

The efficient frontier is calculated as

$$\sigma^2 = \frac{A \cdot \mu^2 - 2B \cdot \mu + C}{\Delta} = a \cdot \mu^2 - b \cdot \mu + c \tag{6}$$

where

$$A = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1} \tag{7}$$

$$B = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \tag{8}$$

$$C = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \tag{9}$$

1 denotes the unity vector, $\boldsymbol{\Sigma}^{-1}$ is the inverted variance covariance matrix, and $\boldsymbol{\mu}$ is the vector of expected returns, calculated as the arithmetical average of all one month returns of each instrument. Δ is the determinant, calculated as

$$\Delta = AC - B^2 \tag{10}$$

For the case with three instruments, including convertible bonds, expected return and the variance covariance matrix are:

Inverted variance covariance matrix:			
	Bonds	Convertibles	Stocks
Bonds	9832.1507	-2781.4723	1918.5632
Convertibles	-2781.4723	2601.2619	-1622.2152
Stocks	1918.5632	-1622.2152	1634.5163

Expected return:	
Bonds	0.0066
Convertibles	0.0046
Stocks	-0.0043

This leads to these factors in the mean variance equation:

a = 9.9369, b = 0.0937, c = 0.0003.

For the case with only straight bonds and stocks, math is similar, leading to these results:

Inverted variance covariance matrix:		
	Bonds	Stocks
Bonds	6857.9834	183.9640
Stocks	183.9640	622.8603

Expected return:	
Bonds	0.0066
Stocks	-0.0043

a = 15.3928, b = 0.1694, c = 0.0006

The first derivation of the two mean variance equations provides the general minimal variance portfolio (or rather, it's return):

with convertibles:

$$\begin{aligned} &\sigma^2 = 9.9369\mu^2 - 0.0937\mu + 0.0003 \\ &f'(\mu) = 19.8738\mu - 0.0937 = 0 \rightarrow \mu_{\min} = 0.4714\% \end{aligned} \quad (11)$$

without convertibles:

$$\begin{aligned} &\sigma^2 = 15.3928\mu^2 - 0.1694\mu + 0.0006 \\ &f'(\mu) = 30.7856\mu - 0.1694 = 0 \rightarrow \mu_{\min} = 0.5503\% \end{aligned} \quad (12)$$

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